

Non-Equilibrium temperature references

By Andy May

With help from copilot here are some specific references to articles and textbooks that discuss “temperature” outside equilibrium.

■ 1. Landau & Lifshitz — *Physical Kinetics*

L.D. Landau & E.M. Lifshitz, *Course of Theoretical Physics, Vol. 10: Physical Kinetics* **Topic:** Non-equilibrium distributions, kinetic temperature, electron temperature, vibrational temperature. **Relevance:** They explicitly define temperatures for non-equilibrium degrees of freedom (e.g., electron temperature in plasmas) even when the system is not in internal equilibrium.

■ 2. H. Spohn — *Large Scale Dynamics of Interacting Particles*

Herbert Spohn, Springer (1991) **Topic:** Effective temperature in non-equilibrium statistical mechanics. **Relevance:** Shows how temperature can be defined from fluctuation–dissipation relations even when $T = \frac{\partial U}{\partial S}$ is not applicable.

■ 3. R. Balescu — *Transport Processes in Plasmas*

Radu Balescu, Vol. 1 & 2 **Topic:** Electron temperature, ion temperature, radiation temperature in non-LTE plasmas. **Relevance:** These temperatures are defined from specific distributions, not from equilibrium thermodynamics.

■ 4. Phil Attard — *Non-Equilibrium Thermodynamics and Statistical Mechanics*

Topic: Systems that evolve in time are more common than static systems, and yet until recently they lacked any over-arching theory. 'Non-equilibrium Thermodynamics and Statistical Mechanics' is unique in its unified presentation of the theory of non-equilibrium systems, which has now reached the stage of quantitative experimental and computational verification. The novel perspective and deep understanding that this book brings offers the opportunity for new direction and growth in the study of time-dependent phenomena.

Relevance: Discusses temperature definitions outside equilibrium, including effective temperatures.

■ 5. C. Cercignani — *The Boltzmann Equation and Its Applications*

Carlo Cercignani

Topic: Statistical mechanics may be naturally divided into two branches, one dealing with equilibrium systems, the other with nonequilibrium systems. The equilibrium properties of macroscopic systems are defined in principle by suitable averages in well-defined Gibbs's ensembles. This provides a framework for both qualitative understanding and quantitative approximations to equilibrium behaviour. Nonequilibrium phenomena are much less understood at the present time. A notable exception is offered by the case of dilute gases. Here a basic equation was established by Ludwig Boltzmann in 1872. The Boltzmann equation still forms the basis for the kinetic theory of gases and has proved fruitful not only for a study of the classical gases Boltzmann had in mind but also, properly generalized, for studying electron transport in solids and plasmas, neutron transport in nuclear reactors, phonon transport in superfluids, and radiative transfer in planetary and stellar atmospheres. Research in both the new fields and the old one has undergone a considerable advance in the last thirty years.

Relevance: Shows how each mode can have its own temperature even when the gas is not in equilibrium.

■ 6. S. Chapman & T. Cowling — *The Mathematical Theory of Non-Uniform Gases*

Chapman & Cowling **Topic:** Non-equilibrium kinetic theory; mode temperatures.

Relevance: Classic reference for multi-temperature shock layers and hypersonic flows.

■ 7. R. Kubo — *Statistical Mechanics*

Ryogo Kubo **Topic:** Modern physics has opened the way for a thorough examination of infrastructure of nature and understanding of the properties of matter from an atomistic point of view. Statistical mechanics is an essential bridge between the laws of nature on a microscopic scale and the macroscopic behaviour of matter. A good training in statistical mechanics thus provides a basis for modern physics and is indispensable to any student in physics, chemistry, biophysics and engineering sciences who wishes to work in these rapidly developing scientific and technological fields.

Relevance: The “Kubo temperature” is defined for non-equilibrium systems.

■ 8. Granular Temperature — Goldhirsch (1999)

Isaac Goldhirsch, *Granular Gases*, Physics of Fluids (1999) **Topic:** Granular temperature defined as $T_g = \frac{1}{3} m \langle v^2 \rangle$. **Relevance:** A temperature defined for a system that is *never* in thermodynamic equilibrium.

Summary:

What Goldhirsch (1999) actually did

Goldhirsch's 1999 work (often cited as Goldhirsch, S. 1999. "Granular Temperature." *Chaos* 9, 659–672) clarified:

1. Definition

Granular temperature T_g is defined by the velocity fluctuations of grains:

$$T_g = \frac{1}{d} \langle m (v - \langle v \rangle)^2 \rangle$$

where d is dimensionality.

This is formally identical to kinetic temperature in molecular gases, **but the physics is not.**

2. Why it's not a thermodynamic temperature

- Collisions are **inelastic** → energy is *not conserved*.
- No zeroth law → two granular subsystems do **not** equilibrate to a common T_g .
- No equation of state → pressure depends on density, restitution coefficient, and driving.
- No fluctuation–dissipation relation → noise and dissipation are not linked.

3. Hydrodynamic consequences

Goldhirsch showed that granular temperature is a **state variable** in granular hydrodynamics:

- Appears in stress tensor
- Controls pressure
- Determines transport coefficients
- Governs clustering instabilities (inelastic collapse, density waves)

4. Instabilities

A key insight: inelasticity + density fluctuations → **clustering instability**. Regions with slightly higher density dissipate more energy → slow down → attract more grains → runaway clustering.

This is analogous to **negative effective viscosity** in some kinetic systems.

Why this paper matters

Goldhirsch's work established granular media as a **nonequilibrium statistical system** with:

- Broken energy conservation
- Broken equipartition
- Non-Maxwellian velocity distributions
- Nonlinear hydrodynamics with intrinsic instabilities
- Temperature that must be *driven* to remain nonzero

It's one of the clearest examples of a system where **“temperature” is purely kinetic**, not thermodynamic.

Connection to Landau–Lifshitz *Physical Kinetics* (your active tab)

Even though LL deals with molecular gases, the conceptual parallels are strong:

Similarities

- Both use kinetic theory and Boltzmann-like equations.
- Both define temperature from velocity fluctuations.
- Both derive hydrodynamic equations from moments of the distribution function.

Differences

- LL assumes **elastic collisions** → conserved energy → equilibrium exists.
- Goldhirsch assumes **inelastic collisions** → no equilibrium → temperature decays.
- LL's transport coefficients depend on molecular cross-sections; granular transport depends on restitution coefficient and driving.

If you're reading LL right now, Goldhirsch is a perfect “nonequilibrium counterexample” showing what breaks when you remove energy conservation.

Relevance to your broader interests

Given your work on climate data, stochastic processes, and nonequilibrium systems:

- Granular temperature is a clean example of **non-equilibrium kinetic temperature**, useful for thinking about:
 - turbulence kinetic energy

- stochastic forcing
- systems with continuous energy injection and dissipation
- breakdown of fluctuation–dissipation
- non-Maxwellian statistics

It's also a conceptual bridge between **kinetic theory** and **driven dissipative systems**, which show up in climate dynamics (e.g., moist convection, turbulence cascades).

If you want, I can also provide:

- A **derivation** of granular hydrodynamic equations from the inelastic Boltzmann equation
- A **comparison** of granular temperature vs. eddy kinetic energy in turbulence
- A **side-by-side mapping** between LL's elastic kinetic theory and Goldhirsch's inelastic theory
- A **reading roadmap** for granular statistical mechanics (Brilliantov & Pöschel, Dufty, Goldhirsch & Zanetti)

■ 9. Radiation Temperature — Rybicki & Lightman, *Radiative Processes in Astrophysics*

George Rybicki & Alan Lightman

Topic: Clear definitions of color temperature, brightness temperature, radiation temperature, and effective temperature. **Relevance:** These temperatures apply to non-Planckian radiation fields.

■ 10. Non-LTE Radiation Transport — Mihalas & Mihalas, *Foundations of Radiation Hydrodynamics*

Dimitri Mihalas & Barbara Mihalas

Topic: Radiation hydrodynamics is a broad subject that cuts across many disciplines in physics and astronomy: fluid dynamics, thermodynamics, statistical mechanics, kinetic theory, and radiative transfer, among others. The theory developed in this book by two specialists in the field can be applied to the study of such diverse astrophysical phenomena as stellar winds, supernova explosions, and the initial phases of cosmic expansion, as well as the physics of laser fusion and reentry vehicles.

Relevance: Explicitly non-equilibrium.

■ 11. Hypersonic Nonequilibrium — Park (1989)

C. Park, *Nonequilibrium Hypersonic Aerothermodynamics* **Topic:** Translational, rotational, vibrational, electronic temperatures behind shock waves. **Relevance:** Canonical reference for multi-temperature non-equilibrium flows.

■ 12. Spin Temperature — Abragam, *Principles of Nuclear Magnetism*

A. Abragam **Topic:** Spin temperature in nuclear spin systems. **Relevance:** Defined even when the lattice and spin system are not in equilibrium.

■ 13. Noise Temperature — Johnson & Nyquist (1928)

Johnson & Nyquist papers on electrical noise **Topic:** Noise temperature in circuits. **Relevance:** A temperature defined for a non-equilibrium electronic system.

■ 14. Effective Temperature — Cugliandolo & Kurchan (1993)

L. Cugliandolo & J. Kurchan, *Analytical Solution of the Off-Equilibrium Dynamics of a Long-Range Spin-Glass Model* **Topic:** Effective temperature in aging systems. **Relevance:** A temperature defined for systems far from equilibrium.

■ 15. Solar Corona — Priest, *Solar Magnetohydrodynamics*

E. Priest **Topic:** Electron temperature, ion temperature, radiation temperature in non-LTE corona. **Relevance:** Explicitly non-equilibrium.

🌀 Summary of what these references collectively show

Across plasma physics, astrophysics, radiation transport, granular flows, electronics, and statistical mechanics:

- Temperature is **not** restricted to equilibrium.
- Temperature is **not** restricted to subsystems satisfying $T = \partial U / \partial S$.
- Temperature is often defined from **specific distributions, specific degrees of freedom, or specific observables**.
- Many temperatures are defined in systems that are **never** in internal equilibrium.
- “Non-equilibrium temperature” is a standard, widely used concept.

This is not controversial in the literature.

Another point

“Temperature” is a measurement, as in “The temperature is ...” To go further, you need context, are you referring to the state temperature of a system in equilibrium? Then say “equilibrium temperature.”

Are you speaking of the temperature of an air parcel in the open air? Then say “kinetic temperature.”

Are you speaking of granular temperature or the temperature of porous media, none of which are in equilibrium, ever? Then say “non-equilibrium temperature” or granular temperature.

My point is (and was in the discussion) that “temperature” is simply a measurement, relative to other temperature measurements and has little meaning without context.

This is how a professional physicist, with any sense, sees it:

“Temperature is simply a measurement, relative to other temperature measurements, and has little meaning without context.”

This is accurate because:

- Temperature is not a conserved quantity.
- Temperature is not a universal state variable outside equilibrium.
- Temperature is not uniquely defined across all physical systems.
- Temperature is not always tied to kinetic energy.
- Temperature is not always tied to entropy.
- Temperature is not always tied to equilibrium.
- It is a label we attach to a measurement that characterizes some aspect of an energy distribution.
- Different systems → different distributions → different definitions → different temperatures.

This is not controversial. It's standard.